

Female Education and Employment: Making the Most of Talents*

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This draft: May 2009

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Abstract

We develop a general equilibrium OLG growth model where women are heterogeneous with respect to the child care costs they bear, there is imperfect information along this dimension and education decisions are endogenous. We show that there is a number of women who have invested in education and find it profitable not to enter the labour market after giving birth to a baby. Their non-participation generates a waste of talent, since the effective output is lower than the potential one as determined by human capital investment. We determine the growth rate of human capital and output and analyse how they depend on the institutional and cultural environment. As a characterisation of the institutional environment we study a tax-transfer scheme that can support the education and participation decision and analyse its emergence in a political game. We show that this policy has positive repercussions on growth.

Keywords: child care costs, asymmetric information, growth, tax-transfer scheme, majority voting.

JEL Classification: J16, J24.

1 Introduction

Industrialized countries are characterized by large cross-country differences in terms of female participation and employment rates. The same can be said if one focuses on education and looks at the share of women with upper secondary education. While for instance in Italy only 48% of women in the 25-64 cohort had at least upper secondary education in 2004, in Sweden they were 85%. If we focus on younger cohorts this gap is smaller but not yet closed. Not only are there differences in terms of the share of educated women, but also in terms of the percentages of educated women who work, indicating that female human capital is often underutilized. Among women with at least upper secondary education, 61% of Spanish women works, while more than 74% of women works in countries such as Denmark, Sweden, the Netherlands and the UK.

Figure 1 and 2 suggest that countries where the number of educated women and the percentages of educated women who work are higher are those where public subsidies to home produced goods (child care for instance) are larger, or where expenditures on family policies account for a greater share of GDP or where measures of flexibility on the labor market are more used (part-time for instance).¹ Scandinavian countries are an example. These are also the countries where the attitudes to female work on the market are the most positive. In Figure 1d and 2d we build an index of gender culture based on data from the World Value Survey (1999)² to measure the attitudes of individuals towards female work

¹In Figure 1a we plot the share of female with at least upper secondary education in 2004 (OECD, 2006) and the availability of part-time jobs (same year) based on Eurostat data. In Figure 1b we plot the share of female with at least upper secondary education and the final child care score for children of age 0-2 taken from De Henau et al. (2007). In Figure 1c we plot the share of female with at least upper secondary education and family expenditure as a percentage of GDP taken from the OECD Family Database and referred to 2003. In Figure 2a, 2b and 2c on the vertical axis we have the share of educated women who are employed in 2004 (OECD, 2006), instead of the share of educated women. These are simple correlations and do not imply any causal relationship.

²The culture index for each country is obtained considering the answers to the following three questions asked by the WVS (1999): "When jobs are scarce, men should have more right to a job than women (C001)"; "A working mother can establish just as a warm and secure relationship with her children as a mother who does not work (D056)" and "Being housewife is just as fulfilling as working for a pay (D057)".

on the market. In countries where our index of gender culture is higher, people are more favorable to female employment, and the share of women with at least upper secondary education as well as the employment of the educated women are higher.³

Previous contributions have studied the impact of cross-country differences in institutions on the female labour market participation and on the fertility decision (see Del Boca et al. (2007) and De Henau et al. (2007) for instance). The relationship between institutions and educational choices is instead less explored. By the same token, Fernandez (2007) and Fernandez and Fogli (2007) for instance study the role of culture on female employment and fertility, while less attention has been devoted to the influence that cultural variables may have on the human capital investment decision. Including educational choices into the analysis of female employment is however essential to shed a light on the unused human capital of educated women who do not work and to analyze what can be done to gain these potential resources.

Why are there women who invest in education but do not participate in the labor market? Why do different countries show different performances in terms of education and participation? What are the implications for growth of the female absence from the labour market?

We propose an explanation whose main ingredients are the heterogeneity of women with respect to the child care costs they bear, the imperfect information along this di-

For the first statement, we assign a value of 1 to people who agree and a value of 2 to people who disagree. For the second and third statement, we assign a value of 1 to people who agree strongly, 2 to people who agree, 3 to people who disagree and 4 to people who strongly disagree. We then divide the sum so obtained for each question by the number of answers given in order to have an average score. The index is finally obtained summing the average score for the first and third question and subtracting the score for the second question, so that a higher value of the index indicates a higher acceptance of female employment, i.e. a higher gender culture.

³While the focus here is on the role of institutions and culture, how good the labour market is in terms of rewarding human capital investments can contribute to explain the cross-country differences in female education and employment. This explanation is however silent on the existing cross-country differences in the gaps in employment rates of women with/without children: while the employment rates of women with children are typically lower than the ones of women without children, for instance in France or Scandinavian countries this gap is smaller than in Italy or Germany.

mension and the endogeneity of the education decision. Child care costs depend both on individual characteristics and on the institutional or cultural environment where the woman lives. As to the individual component, some women may bear a higher individual cost than others, i.e. a higher amount of required homecare. This heterogeneity may depend on several factors such as how demanding the kid is, due for instance to her health status or good nature; the presence of family or informal networks which can help taking care of the kid; how supportive the partner is or how tight the after-birth relationship between mother and kid turns out to be. We consider information about this cost to be imperfect at the time of the education decision and revealed only after the child's birth. As to the institutional and cultural component, women living in certain areas may face a more favorable institutional environment, for instance because the legislation provides instruments for work-life balance and for a flexible labour market, incentives to the support from the partner, premia to the firms which promote women, or because public or private day-care services are available. They may also live in a cultural environment where female market work is promoted, accepted or at least not opposed. While the cultural attitude towards women is assumed to be known and given throughout the paper, institutions will first be treated as given (in the same way as culture) and then they will be voted upon.

We build a two-period general equilibrium overlapping generations model with endogenous growth. Women are heterogeneous in talent and in child care costs. Based on their expectations on the child care cost, in the first period they decide how much time to devote to human capital accumulation. In the second period, they give birth to a child, their true cost is revealed and they decide whether to work or not.

We identify a threshold level of ability such that only women whose ability is above such threshold find it convenient to invest in education. This cut-off is increasing in the average child care cost. We show that, after the child birth, there is a number of women who have invested in education who find it profitable not to enter the labour market, knowing they are high-cost type. We discuss the role that the institutional and cultural environment have on the number of educated women and on the likelihood of their participation. The decision not to participate to the labour market has macroeconomic implications: effective

output is lower than the potential one as determined by human capital investment and so is the growth rate. As a characterisation of the institutional environment, we then propose a tax-transfer scheme targeted to working women and show that its implementation has positive repercussions on education, participation and growth. We finally show under what conditions this scheme can emerge as the equilibrium of a majoritarian voting game.

Many previous contributions have analyzed the role of human capital as a fundamental engine for growth (see the seminal contribution by Lucas, 1988 and Glomm and Ravikumar, 1992). Closer to our question, some authors have studied the role of female human capital and gender equality in education as determinants of growth. Using a large cross-section of countries, Dollar and Gatti (1999) find that not investing in female education is a non efficient economic choice, mainly due to religion and civil freedom indicators, and this inefficiency is a more significant distortion at higher levels of income. Cavalcanti and Tavares (2007) calculate that the output cost of gender discrimination is sizeable. One of the channels which delivers this result is the high level of fertility associated with gender discrimination. Fertility is also the focus of Lagerloef (2003) and de la Croix and Vander Donckt (2008). Another strand of the recent literature has focused on the negative effects for growth deriving from a misallocation of individuals in jobs, which may arise when social mobility is low. In Galor and Tsiddon (1997) and Hassler and Mora (2000) technological changes may increase social mobility, and thus growth. Bernasconi and Profeta (2007) consider instead the role of public education in reducing the mismatch, thus increasing social mobility and growth. Though our analysis shows some similarities with these previous arguments, none of them focuses on the costs associated with the fact that some educated women do not work.

The issue of the drop out of women from the labour force can also be framed in the recent literature which studies how gender gaps in wage and in participation can arise when the distribution of ability between men and women is the same (Francois, 1998; Albanesi and Olivetti, 2008; Bjerk and Hahn, 2007, Lommerud and Vagstad, 2007). These contributions share a common assumption, namely, skills are given and cannot be accumulated. Our attention here is instead on the female education and participation

decisions and on their impact on growth.

The role of education is the focus of a very recent literature which analyses the effects of pre-marital schooling decisions on the marriage market: educational attainment may influence the marital surplus share that can be extracted in the match, by increasing the prospects of marrying an educated partner and by affecting the competitive strength in the bargaining within the couple (Iyigun and Walsh, 2007b; Chiappori et al., 2008). We here abstract from gains in the marriage market as a motivation to acquire education: as long as childrearing is more costly for women than for men and this difference cannot be completely eliminated by the mother's educational attainment (see Iyigun and Walsh, 2007a for a similar assumption), marriage market motivations would enrich the framework without eliminating our mechanism.

The paper is organized as follows: the next Section develops the model, Section 3 analyses the macroeconomic implications of the individual education and participation decisions under imperfect information; Section 4 introduces the tax-transfer scheme and Section 5 concludes. Proofs are in the Appendix.

2 The model

2.1 General features

We develop a two-period general equilibrium overlapping generations model with endogenous growth. Women are heterogeneous in talent and in child care costs. These costs depend both on individual characteristics ρ and on the institutional and cultural environment I where the woman lives.⁴ Some women may bear a higher cost ρ than others, i.e. a higher amount of required homecare. They may also live in a more favorable environment I , in terms of institutions or cultural attitudes of the society. Formally, $g(\rho, I)$ indicates the child care cost as a share of income, with partial derivatives $g_\rho(\rho, I) > 0$, $g_I(\rho, I) < 0$. While I is common knowledge, information about ρ is imperfect. For tractability, we assume that $g(\rho, I) = \frac{\rho}{I} \leq 1$ and that ρ can take only two values $\rho^H < 1$ with probability π and $\rho^L < 1$ with probability $(1 - \pi)$, where $\rho^H > \rho^L$. Women and firms know π , ρ^H and

⁴See the Introduction for the interpretation of the two cost components.

ρ^L . Women do not know their own cost-type ρ^H or ρ^L until the child's birth. The same holds for firms, which operate in a perfectly competitive environment. As to talent e^i , it is distributed on the interval $[0, 1]$ with continuous density function $f(\cdot)$ and it is known to women and firms.

In their first period of life, women decide how much time to devote to human capital accumulation and how much time to spend in leisure. While women know their talent when deciding their investment in human capital, they form expectations on ρ . In the second period, women give birth to a child and they discover their true type.⁵ Those who invested in human capital enter the labor market and work if they are low-cost types. The high-cost types decide whether to work or not. Those who did not invest in human capital do not enter the labour market⁶ and live out of their family endowment \widehat{w}_{t+1} , which is by assumption the same for everybody.⁷ Non-working mothers enjoy a reduction η of child care cost, indicating that child care is mostly home provided when a woman is out of the labour market and that in-home provision of child care is cheaper. Consumption takes place at the end of the second period of life when agents consume all their lifetime income. The population growth rate is zero.

⁵Notice that we do not endogenize fertility and thus the margins of adjustments are education and participation and not the number of kids.

⁶It is standard to assume that, starting from a distribution of abilities, individuals who do not invest in education remain unskilled and constitute a homogeneous group. In our set-up either all of them, based on their expectation on ρ , find it profitable to work, or none of them find it advantageous to participate to the labour market. We focus on this second case, which implies that the choice of investing in education and working is taken against the alternative of not investing and not working. Since unskilled individual do not contribute to the accumulation of human capital, their participation to the labour market is orthogonal to the questions addressed in this paper.

⁷Assuming that \widehat{w}_{t+1} is the same for everybody implies that investing in education does not increase, for instance, the probability of having a partner with higher income and therefore of having a higher family endowment. This is a conservative assumption in view of the fact that the marriage market is not explicitly analyzed in our model.

2.2 Individual problem

(sposterei queste due righe appena prima dell'equazione 9 e parlerei della expected indirect U function) **OK**

The objective function of woman i is:

$$EU_t^i = \pi c_{t+1}^{iH} + (1 - \pi) c_{t+1}^{iL} - (1 - n_t^i)^\alpha \quad (1)$$

where c_{t+1}^{iH} and c_{t+1}^{iL} are the consumption during the second period of life when agent i is characterized by ρ^H and ρ^L respectively, n_t^i is leisure and $\alpha > 1$. When agent i works, the budget constraints are:

$$c_{t+1}^{iH} = (w_{t+1}^i + \widehat{w}_{t+1}) \left(1 - \frac{\rho^H}{I}\right) \quad (2)$$

$$c_{t+1}^{iL} = (w_{t+1}^i + \widehat{w}_{t+1}) \left(1 - \frac{\rho^L}{I}\right) \quad (3)$$

where w_{t+1}^i is the wage paid to agent i by the competitive firm. Output per worker (**o per woman**) is:

$$y_{t+1}^i = h_{t+1}^i \quad (4)$$

with h_{t+1}^i denoting the individual level of human capital.

Profit maximization by the competitive firms will deliver $w_{t+1}^i = h_{t+1}^i$. To capture that in a dynamic context the family endowment can vary over time, we assume that \widehat{w}_{t+1} is a function of the average level of human capital in the entire economy (including therefore men and women). We rewrite the latter as a linear transformation of the female average level of human capital \bar{h}_{t+1} , i.e. we set $\widehat{w}_{t+1} = \nu \bar{h}_{t+1}$, with $\nu > 0$.

Human capital accumulates according to the following Cobb-Douglas technology:

$$h_{t+1}^i = \vartheta \nu^\delta \bar{h}_t^\delta e^{i\gamma} (1 - n_t^i)^\beta \quad (5)$$

where ϑ is a scale parameter, $\nu \bar{h}_t$ is the average level of human capital of the previous generation, and δ, γ and $\beta \in (0, 1)$ are the parameters of the human capital production function.

When they do not work, women do not accumulate human capital, $n_t^i = 0$ and the budget constraints are:

$$c_{t+1}^H = \widehat{w}_{t+1} \left[1 - \left(\frac{\rho^H}{I} - \eta \right) \right] \quad (6)$$

$$c_{t+1}^L = \widehat{w}_{t+1} \left[1 - \left(\frac{\rho^L}{I} - \eta \right) \right] \quad (7)$$

where $\eta > 0$ indicates a reduction of child care cost for non-working mothers,⁸ implying that child care is mostly home provided when a woman is out of the labour market and that in-home provision of child care is cheaper.

Women choose the amount of human capital investment maximizing equation (1), subject to (2) and (3). The optimal level of investment in human capital for women of talent e^i is given by:

$$1 - n_t^i = \left[\frac{\beta}{\alpha} \vartheta \nu^\delta \overline{h}_t^{-\delta} e^{i\gamma} \left(1 - \frac{\overline{\rho}}{I} \right) \right]^{\frac{1}{\alpha-\beta}} \quad (8)$$

where $\overline{\rho} = \pi \frac{\rho^H}{I} + (1 - \pi) \frac{\rho^L}{I}$ is the average child care cost and where we assume $\alpha > \beta$ to guarantee that talent and education are positively related.

Women decide whether to invest in skill acquisition and work or whether to remain unskilled and out of the labour market comparing their expected indirect utility function in the two cases. The expected indirect utility function of educated and non-educated women are, respectively:

$$U_{t,n<1}^i = \left\{ \vartheta \nu^\delta \overline{h}_t^{-\delta} e^{i\gamma} \left[\frac{\beta}{\alpha} \vartheta \nu^\delta \overline{h}_t^{-\delta} e^{i\gamma} \left(1 - \frac{\overline{\rho}}{I} \right) \right]^{\frac{\beta}{\alpha-\beta}} + \widehat{w}_{t+1} \right\} \left(1 - \frac{\overline{\rho}}{I} \right) + \quad (9)$$

$$- \left[\frac{\beta}{\alpha} \vartheta \nu^\delta \overline{h}_t^{-\delta} e^{i\gamma} \left(1 - \frac{\overline{\rho}}{I} \right) \right]^{\frac{\alpha}{\alpha-\beta}}$$

$$U_{t,n=1}^i = \widehat{w}_{t+1} \left[1 - \left(\frac{\overline{\rho}}{I} - \eta \right) \right] \quad (10)$$

Comparing (9) and (10), we identify the following threshold level Ψ_t of innate talent such that women with $e^i \geq \Psi_t$ invest in education and work:

$$\Psi_t \equiv \frac{(\widehat{w}_{t+1} \eta)^{\frac{\alpha-\beta}{\alpha\gamma}}}{\left\{ \vartheta \nu^\delta \overline{h}_t^{-\delta} \left[\frac{\beta}{\alpha} \vartheta \nu^\delta \overline{h}_t^{-\delta} \left(1 - \frac{\overline{\rho}}{I} \right) \right]^{\frac{\beta}{\alpha-\beta}} \left(1 - \frac{\overline{\rho}}{I} \right) \left(1 - \frac{\beta}{\alpha} \right) \right\}^{\frac{\alpha-\beta}{\alpha\gamma}}} \quad (11)$$

⁸See Bjerck and Hahn (2007) for a similar assumption. Notice that our results would hold also under the assumption that non-working women do not experience any child care cost, $c_{t+1}^H = c_{t+1}^L = \widehat{w}_{t+1}$.

It follows immediately that $\frac{\partial \Psi_t}{\partial \bar{\rho}} > 0$ and $\frac{\partial \Psi_t}{\partial I} < 0$. When the child care cost is higher, either because the average individual cost is higher or the institutional/cultural environment is less favorable, fewer women invest in education and plan to work.

At the beginning of the second period of life, all women give birth to a child and they discover their true cost type. If they are high-cost types $\rho^H > \bar{\rho}$, they reconsider whether to work or not.⁹ Recalling that the investment in education is sunk, women compare the utility they can enjoy in the second period of life in the case they work $U_{t+1,n=0}$ or in the case they do not participate $U_{t+1,n=1}$. Namely, simplifying terms, high-cost women will work when:

$$(h_{t+1}^i + \widehat{w}_{t+1})(1 - \frac{\rho^H}{I}) > \widehat{w}_{t+1}(1 - \frac{\rho^H}{I} + \eta) \quad (12)$$

Following the same procedure as before, we can identify a new threshold level of ability Ψ'_{t+1} such that only high-cost women whose ability $e^i \geq \Psi'_{t+1}$ do indeed work, with:

$$\Psi'_{t+1} \equiv \frac{(\widehat{w}_{t+1}\eta)^{\frac{\alpha-\beta}{\alpha\gamma}}}{\left\{ \vartheta\nu^\delta \overline{h}_t^\delta \left[\frac{\beta}{\alpha} \vartheta\nu^\delta \overline{h}_t^\delta (1 - \frac{\bar{\rho}}{I}) \right]^{\frac{\beta}{\alpha-\beta}} \left(1 - \frac{\rho^H}{I}\right) \right\}^{\frac{\alpha-\beta}{\alpha\gamma}}} \quad (13)$$

Again, it follows straightforwardly that $\frac{\partial \Psi'_{t+1}}{\partial \rho^H} > 0$ and that $\frac{\partial \Psi'_{t+1}}{\partial I} < 0$. When the child care cost is higher, either because the individual cost ρ^H is higher or the institutional and cultural environment is less favorable, the level of ability for which women of type ρ^H find it convenient to work goes up.

When $\Psi'_{t+1} \geq \Psi_t$, a share of educated women find it convenient not to work after the child birth. This happens when the following condition is satisfied:

$$\Psi'_{t+1} \geq \Psi_t \text{ if } \frac{\rho^H}{I} - \frac{\bar{\rho}}{I} \left(1 - \frac{\beta}{\alpha}\right) \geq \frac{\beta}{\alpha} \quad (14)$$

Remark 1 *A more favorable institutional or cultural environment reduces the likelihood of observing educated women who do not participate to the labour market. Indeed, ceteris paribus, an increase in I makes it more difficult to satisfy (14).*

⁹As to those who discover to be low-cost type, if they invested, their incentive to participate is not affected by the revelation of their own true type. If they did not invest, they did not accumulate any human capital, as equation (5) shows, and therefore they never have access to the labour market. See also footnote 4.

If the above condition is satisfied, the number of educated women who do not participate to the labour market at time $t + 1$ is:

$$Q_{t+1} = \pi \left[F(\Psi'_{t+1}) - F(\Psi_t) \right] \quad (15)$$

In the rest of the paper we concentrate on the case where $Q_{t+1} > 0$, which is the focus of our interest.

Notice that our model is characterized by two types of market failures: the presence of the externality and of imperfect information. While it is clear that the presence of the externality is such that agents invest suboptimally in human capital, the impact of imperfect information on human capital accumulation is less straightforward. Indeed, on the one side women with low child care costs invest in education less than they would have, had they known right from start their true-cost type. However, high cost types invest more under imperfect information than they would under the assumption that information is perfect.¹⁰ What certainly holds is that in the economy with perfect information we do not observe educated women who do not participate to the labor market. Indeed, the non participation of high-cost educated women is generated by imperfect information. While imperfect information cannot be completely eliminated, its impact can be attenuated by stronger institutions or more favorable attitudes. Institutions can be interpreted as an insurance device against the risk of being high-cost women: indeed, they would otherwise bear very high costs of individual child care. When I is large, the differences between being a high-type and being a low-type become irrelevant and therefore all women who have invested in education do indeed work after the child birth. In other words, a level of I not large enough can be interpreted as the real responsible for non participation.¹¹ However, institutions are not exogenous. In Section 4 we represent I through a tax-transfer

¹⁰We return on this point in the next section where we identify a sufficient condition for observing underinvestment in human capital in the economy with imperfect information.

¹¹As it should be clear, we have abstracted from the explicit modelling of the male education and work decision. If male decisions were to be introduced, we would consider a context where, for given institutional environment, men are homogeneous with respect to the child care cost ρ^M which enters both their education and participation decision. This implies that there are not any educated men who would decide not to participate due to the child care costs, differently from women. This is in line with the evidence that - on average - male labour supply is less elastic than female. The comparison between the female and male

scheme and study how it affects the education and participation decision. We then analyze whether a scheme of this type can be the equilibrium of a politico-economic game.

3 The macroeconomy

In this Section we analyze the macroeconomic implications of the non participation of educated women.¹²

Aggregate human capital formed at t is:

$$H_t = \int_{\Psi_t}^1 h_t f(e) de \quad (16)$$

Aggregate output at $t + 1$ is given by¹³

$$Y_{t+1} = (1 - \pi) \int_{\Psi_t}^1 h_t f(e) de + \pi \int_{\Psi'_{t+1}}^1 h_t f(e) de. \quad (17)$$

We define the loss L_{t+1} which the economy experiences at $t + 1$ as the difference between the human capital accumulated and output produced,¹⁴ as a share of aggregate human capital:

$$L_{t+1} = \frac{H_t - Y_{t+1}}{H_t} \quad (18)$$

It measures the reduction in potential output which an economy suffers when educated education levels would depend, among other things, on the relationship between ρ^M and $\bar{\rho}$. When they are close, male and female take a similar education decision, other things constant. This does not imply that they have the same behavior on the labour market as, at the child birth, the ex ante imperfect information for women counts.

¹²Notice that we are here focusing on the contributions that only women give to aggregate human capital and output.

¹³It is straightforward to show that $\pi \geq \frac{1}{2}$ is a sufficient condition for observing a higher level of human capital and output in the economy with perfect information where women know from the start their own type ρ^L and ρ^H rather than in the economy where the true cost is revealed only after child birth. Notice also that in the case of perfect information there is no difference between aggregate human capital and output because all educated women will work.

¹⁴We recall that unskilled workers, i.e. those who did not invest in human capital, are not producing any output in our framework.

women do not work.¹⁵ Using (16) and (17), simple algebra delivers the following expression for the loss:

$$L_{t+1} = \frac{\pi \int_{\Psi_t}^{\Psi'_{t+1}} h_t f(e) de}{\int_{\Psi_t}^1 h_t f(e) de} \quad (19)$$

3.1 The dynamics

We here make the simplifying assumption that agents are homogeneous with respect to their talent. In this case, (16) and (17) can be rewritten as follows:

$$H_t = [1 - F(\Psi_t)]h_t \quad (20)$$

$$Y_{t+1} = \left\{ 1 - F(\Psi_t) - \pi[F(\Psi'_{t+1}) - F(\Psi_t)] \right\} h_t \quad (21)$$

How H_t and Y_{t+1} evolve over time depend on the behavior of Ψ_t , Ψ'_{t+1} and h_t . Starting from the latter and plugging (8) into (5), the rule of accumulation of per capita human capital is:

$$h_{t+1} = Ah_t^{\frac{\alpha\delta}{\alpha-\beta}} \quad (22)$$

where $A = \left[\frac{\beta}{\alpha} (1 - \frac{\bar{\rho}}{I}) \right]^{\frac{\beta}{\alpha-\beta}} \vartheta^{\frac{\alpha}{\alpha-\beta}} e^{\frac{\alpha\gamma}{\alpha-\beta}} \nu^{\frac{\alpha\delta}{\alpha-\beta}} > 0$ is a constant. Notice, in particular, that $A = A(I)$, with $\frac{\partial A(I)}{\partial I} > 0$.

Starting from the value of the threshold Ψ_t as given by equation (11), we substitute in the numerator $\widehat{w}_{t+1} = \nu \bar{h}_{t+1} = \nu A \bar{h}_t^{\frac{\alpha\delta}{\alpha-\beta}}$ and we rewrite:

$$\Psi_t = \frac{(\eta\nu A)^{\frac{\alpha-\beta}{\alpha\gamma}}}{\vartheta^{\frac{1}{\gamma}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha\gamma}} (1 - \frac{\bar{\rho}}{I})^{\frac{1}{\gamma}} \left(1 - \frac{\beta}{\alpha}\right)^{\frac{\alpha-\beta}{\alpha\gamma}} \nu^{\frac{\delta}{\gamma}}} \quad (23)$$

As to Ψ'_{t+1} , using (13) and (22), we obtain:

$$\Psi'_{t+1} = \frac{(\eta\nu A)^{\frac{\alpha-\beta}{\alpha\gamma}}}{\vartheta^{\frac{1}{\gamma}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha\gamma}} (1 - \frac{\bar{\rho}}{I})^{\frac{\beta}{\alpha\gamma}} \left(1 - \frac{\rho^H}{I}\right)^{\frac{\alpha-\beta}{\alpha\gamma}} \nu^{\frac{\delta}{\gamma}}} \quad (24)$$

¹⁵We are not explicitly accounting for the benefits that human capital investment provides when not employed on the market. This is not to say that human capital investments by non-working mothers do not have positive effects on the economy (for instance through private benefits to children or through externalities to the society as a whole). However, as long as working on the market does not completely crowd out these effects, there are further benefits (or lower losses) to be reaped when educated women work on the market. On the effects of maternal employment on children's human capital see Bernal (2008).

Notice that (23) and (24) do not depend on $h_t = \bar{h}_t$.

Given (23) and (24), the thresholds are constant ($\Psi_t = \Psi_{t+1} = \Psi$ and $\Psi'_t = \Psi'_{t+1} = \Psi'$).

The number of educated women who do not participate to the labour market is constant over time. Thus, human capital and output grows at the same rate. The growth rates of aggregate human capital $g_H = \frac{H_{t+1}}{H_t}$ and output $g_Y = \frac{Y_{t+1}}{Y_t}$ in equations (20) and (21) are entirely determined by the growth rate of per capita human capital $g_h = \frac{h_{t+1}}{h_t}$ with, using (22):

$$g_h = g_H = g_Y = g = Ah_t^{\frac{\alpha\delta}{\alpha-\beta}-1} \quad (25)$$

Proposition 2 (i) If $\frac{\alpha\delta}{\alpha-\beta} \neq 1$, then there exists a unique steady state given by $h_s > 0$ such that $h_{t+1} = h_s$ whenever $h_t = h_s$; (ii) If $\frac{\alpha\delta}{\alpha-\beta} = 1$ and $A(I) \neq 1$ then there does not exist a steady state.

The long run growth rates are characterized as follows: (i) If $\frac{\alpha\delta}{\alpha-\beta} < 1$, then $\lim_{t \rightarrow \infty} h_{t+1}/h_t = 1$; (ii) If $\frac{\alpha\delta}{\alpha-\beta} > 1$, then h_{t+1}/h_t is greater than 1 and increasing over time if the initial level of human capital h_0 is such that $h_0 > h_s$; (iii) If $\frac{\alpha\delta}{\alpha-\beta} = 1$, then there is endogenous growth and $g = h_{t+1}/h_t = A(I)$.

Proof. See Figure 3. ■

Notice that $\frac{\alpha\delta}{\alpha-\beta}$ is the key parameter that determines growth in our model. That the parameters of the Cobb-Douglas human capital technology are crucial to characterize the growth rates is standard in the literature (see Glomm and Ravikumar, 1992). In particular, we find that when $\frac{\alpha\delta}{\alpha-\beta} < 1$ (decreasing returns), the steady state is globally stable and independent of the initial stock of human capital. In this case the long-run growth rate is zero. When $\frac{\alpha\delta}{\alpha-\beta} > 1$ (increasing returns), the steady state is unstable and we have unbounded growth depending on the initial conditions. Finally, when $\frac{\alpha\delta}{\alpha-\beta} = 1$ (constant returns) the growth rate is constant and its level depends on I , as stated in the following corollary.

Corollary 3 *Economies with more favorable cultural and institutional environments are characterized by higher growth rates.*

Proof. It follows straightforwardly from $\frac{\partial A(I)}{\partial I} > 0$. ■

Finally, we focus on the dynamics of the loss L_{t+1} . Using (16) and (21), from equation (19), it is straightforward to see that:

$$\frac{L_{t+1}}{L_t} = 1 \tag{26}$$

i.e. the amount of human capital which does not transform into output (as a share of aggregate human capital) is constant over time and it does not cancel out as the economy grows. For the case when there is endogenous growth we can describe the impact of institutions and culture on the loss as follows.

Corollary 4 *If $\delta = \gamma$ and f is uniform on the interval $[0, 1]$, economies with more favorable cultural and institutional environments are characterized by lower losses of human capital investment.*

Proof. See Appendix (part A). ■

4 The tax-transfer scheme

As a characterisation of the institutional environment, in this section we propose a policy based on a tax-transfer scheme and we study its role in affecting the education and participation decisions. Our interest is in identifying an instrument which the government can use to reduce the waste of talent.¹⁶ I will still capture cultural attitudes. The scheme we analyze requires the payment of proportional contributions τ levied on wages and on family endowments of the entire population. Assuming that not only women but also the government can perfectly observe the women's true cost-type at child birth,¹⁷ the benefits are paid out to working mothers as a proportional discount φ on the cost of child care $\frac{\rho^H}{I}$

¹⁶In other words, we are not looking for the optimal government policy.

¹⁷An alternative would be to assume that the government cannot perfectly observe the true cost-type of women and it pays benefits as a proportional discount φ on the average cost of child care $\bar{\frac{\rho^H}{I}}$. We study this case in Part C of the Appendix.

and $\frac{\rho^L}{I}$.¹⁸ The budget constraint of the scheme is as follows:

$$\begin{aligned} & \tau_{t+1} \left[\pi \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + (1 - \pi) \int_{\Psi_t}^1 w_{t+1}^i f(e) de + \widehat{w}_{t+1} \int_0^1 f(e) de \right] \quad (27) \\ = & \pi \varphi_{t+1} \frac{\rho^H}{I} \left[\int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + \widehat{w}_{t+1} \int_{\Psi'_{t+1}}^1 f(e) de \right] + \\ & (1 - \pi) \varphi_{t+1} \frac{\rho^L}{I} \left[\int_{\Psi_t}^1 w_{t+1}^i f(e) de + \widehat{w}_{t+1} \int_{\Psi_t}^1 f(e) de \right] \end{aligned}$$

where we recall that $w_{t+1}^i = h_{t+1}^i$. The budget constraints of the individual problem change as follows. When agent i invests and works, the budget constraints are:

$$c_{t+1}^{iH} = (w_{t+1}^i + \widehat{w}_{t+1}) \left(1 - \frac{\rho^H}{I} (1 - \varphi_{t+1}) - \tau_{t+1} \right) \quad (28)$$

$$c_{t+1}^{iL} = (w_{t+1}^i + \widehat{w}_{t+1}) \left(1 - \frac{\rho^L}{I} (1 - \varphi_{t+1}) - \tau_{t+1} \right) \quad (29)$$

Women who do not work do not accumulate human capital, $n_t^i = 0$ and the budget constraints are:

$$c_{t+1}^H = \widehat{w}_{t+1} \left[1 - \left(\frac{\rho^H}{I} - \eta \right) - \tau_{t+1} \right] \quad (30)$$

$$c_{t+1}^L = \widehat{w}_{t+1} \left[1 - \left(\frac{\rho^L}{I} - \eta \right) - \tau_{t+1} \right] \quad (31)$$

that is, they contribute to the scheme without being entitled to receiving any benefit.

In order to see how the tax-transfer scheme affects the decision to invest in education and to work, we calculate the new indirect utility functions in the two cases. Equation (9) and (10) can be rewritten as follows:

$$\begin{aligned} U_{t,n < 1}^i &= \left\{ \vartheta \nu^\delta \overline{h}_t^\delta e^{i\gamma} \left[\frac{\beta}{\alpha} \vartheta \nu^\delta \overline{h}_t^\delta e^{i\gamma} \left(1 - \frac{\overline{\rho}}{I} (1 - \varphi_{t+1}) - \tau_{t+1} \right) \right]^{\frac{\beta}{\alpha - \beta}} + \widehat{w}_{t+1} \right\} \\ & \left(1 - \frac{\overline{\rho}}{I} (1 - \varphi_{t+1}) - \tau_{t+1} \right) - \left[\frac{\beta}{\alpha} \vartheta \nu^\delta \overline{h}_t^\delta e^{i\gamma} \left(1 - \frac{\overline{\rho}}{I} (1 - \varphi_{t+1}) - \tau_{t+1} \right) \right]^{\frac{\alpha}{\alpha - \beta}} \quad (32) \end{aligned}$$

$$U_{t,n=1}^i = \widehat{w}_{t+1} \left(1 - \left(\frac{\overline{\rho}}{I} - \eta \right) - \tau_{t+1} \right) \quad (33)$$

¹⁸The proportional discount can be seen as a partial allowance on the child care costs born by the woman. Publicly subsidized child care is widespread, although the degree of generosity and the design of the subsidies vary remarkably across countries.

The new threshold level of ability such that women find it profitable to invest is:

$$\Psi_t^\tau \equiv \frac{\left[\widehat{w}_{t+1}(\eta - \varphi_{t+1}\frac{\bar{p}}{I})\right]^{\frac{\alpha-\beta}{\alpha\gamma}}}{\left\{\vartheta\nu^\delta\bar{h}_t^\delta \left[\frac{\beta}{\alpha}\vartheta\nu^\delta\bar{h}_t^\delta \left(1 - \frac{\bar{p}}{I}(1 - \varphi_{t+1}) - \tau_{t+1}\right)\right]^{\frac{\beta}{\alpha-\beta}} \left(1 - \frac{\bar{p}}{I}(1 - \varphi_{t+1}) - \tau_{t+1}\right) \left(1 - \frac{\beta}{\alpha}\right)\right\}^{\frac{\alpha-\beta}{\alpha\gamma}}}. \quad (34)$$

In order to establish whether the tax transfer scheme is capable of providing stronger incentives to human capital investment, we compare Ψ_t with Ψ_t^τ . The numerator in (11) is always higher than the numerator in (34). By looking at the denominators, a sufficient condition to obtain $\Psi_t > \Psi_t^\tau$ is:

$$\tau_{t+1} \leq \frac{\varphi_{t+1}\bar{p}}{I} \quad (35)$$

which, using the budget constraint in (27), is always satisfied (See Appendix, part B). As the threshold level of ability such that women find it convenient to invest in education is smaller in the presence of the tax-transfer scheme if (35) is satisfied, the tax-transfer system induces more people to acquire human capital.

We now turn to the implications of the presence of the tax-transfer system on the decision to participate. The new threshold level of ability $\Psi_{t+1}^{\prime\tau}$ such that only women whose ability $e^i \geq \Psi_{t+1}^{\prime\tau}$ do indeed work is:

$$\Psi_{t+1}^{\prime\tau} \equiv \frac{\left[\widehat{w}_{t+1}(\eta - \varphi_{t+1}\frac{\bar{p}}{I})\right]^{\frac{\alpha-\beta}{\alpha\gamma}}}{\left\{\vartheta\nu^\delta\bar{h}_t^\delta \left[\frac{\beta}{\alpha}\vartheta\nu^\delta\bar{h}_t^\delta \left(1 - \frac{\bar{p}}{I}(1 - \varphi_{t+1}) - \tau_{t+1}\right)\right]^{\frac{\beta}{\alpha-\beta}} \left(1 - \frac{\rho^H}{I}(1 - \varphi_{t+1}) - \tau_{t+1}\right)\right\}^{\frac{\alpha-\beta}{\alpha\gamma}}}. \quad (36)$$

The following proposition suggests that the tax-transfer system may make it less likely that women who have invested in education decide not to participate. In other words, there exists a range of parameter values under which some of the educated women abandon the labour market in the economy without tax and transfers, but they do not in the economy with tax and transfers.

Proposition 5 *If $\frac{\beta}{\alpha} \leq \frac{\rho^H}{I} - \frac{\bar{p}}{I}(1 - \frac{\beta}{\alpha}) \leq \frac{\beta}{\alpha} \frac{1-\tau_{t+1}}{1-\varphi_{t+1}}$ in the economy without tax and transfers there are women who have invested in education but do not participate, while in the economy with the tax-transfer scheme all women who have invested in education work.*

Proof. Comparing equation (34) and (36), the condition such that $\Psi'_{t+1} > \Psi_t$ is $\frac{\rho^H}{I} - \frac{\bar{\rho}}{I}(1 - \frac{\beta}{\alpha}) > \frac{\beta}{\alpha} \frac{1 - \tau_{t+1}}{1 - \varphi_{t+1}}$. The result comes from $\tau_{t+1} < \varphi_{t+1}$, which follows from (35), and from condition (14). ■

The increase in the number of educated women and in their participation has repercussions on the growth rate of the economy. In the presence of taxes and subsidies to working women the growth rate of the economy is

$$A^\tau = \left[\frac{\beta}{\alpha} \left(1 - \frac{\bar{\rho}}{I}(1 - \varphi_{t+1}) - \tau_{t+1} \right) \right]^{\frac{\beta}{\alpha - \beta}} \vartheta^{\frac{\alpha}{\alpha - \beta}} e^{\frac{\alpha\gamma}{\alpha - \beta}} \nu^{\frac{\alpha\delta}{\alpha - \beta}} \quad (37)$$

which, using (35), is such that $A < A^\tau$, implying that the growth rate of the economy is higher when female education and work are subsidized.

4.1 Voting on τ

In this Section we study whether the tax-transfer scheme described in the previous section can be supported by a majority voting coalition. We assume that individuals vote over their preferred level of the tax rate τ_{t+1} in the first period of life, before knowing their true type and having taken the participation decision.¹⁹ We first observe that agents who do not invest in education and therefore do not work (i.e. all $e^i < \Psi^\tau$) prefer $\tau_{t+1} = 0$ and thus $\varphi_{t+1} = 0$. Agents who have invested in education (i.e. all $e^i \in (\Psi^\tau, 1)$) and will potentially work choose their favorite level of τ_{t+1} by maximizing their indirect utility function (32). We have:

$$\frac{\partial U_{t,n < 1}^i}{\partial \tau_{t+1}} = \left\{ \left(\vartheta \nu^\delta \bar{h}_t^\delta e^{i\gamma} \right)^{\frac{\alpha}{\alpha - \beta}} \frac{\alpha}{\alpha - \beta} \left(1 - \frac{\bar{\rho}}{I}(1 - \varphi_{t+1}) - \tau_{t+1} \right)^{\frac{\beta}{\alpha - \beta}} \frac{\beta}{\alpha} \frac{\beta}{\alpha - \beta} \left(1 - \frac{\beta}{\alpha} \right) + \hat{w}_{t+1} \right\} \left(\frac{\bar{\rho}}{I} \frac{\partial \varphi_{t+1}}{\partial \tau_{t+1}} - 1 \right)$$

Given that the first term in curly brackets is always positive, the sign of $\frac{\partial U_{t,n < 1}^i}{\partial \tau_{t+1}}$ depends on the sign of $\left(\frac{\bar{\rho}}{I} \frac{\partial \varphi_{t+1}}{\partial \tau_{t+1}} - 1 \right)$. Notice that, for $\tau_{t+1} \in [0, \varphi_{t+1} \frac{\bar{\rho}}{I}]$, - see condition (35) - $\frac{\partial U_{t,n < 1}^i}{\partial \tau_{t+1}}$ is always greater or equal to 0. This guarantees that preferences are single-peaked over τ_{t+1} in this interval. It is immediate to show that $\tau_{t+1} = \varphi_{t+1} \frac{\bar{\rho}}{I}$ is a maximum for $\frac{\partial U_{t,n < 1}^i}{\partial \tau_{t+1}}$

¹⁹This, by construction, emphasizes the role of education on the support of the tax and transfer scheme.

As an alternative, we can assume that women vote after their working decision has been taken. We return on this point later momentarily.

for all $e^i \in (\Psi, 1)$. In this case $\frac{\partial U_{t,n < 1}^i}{\partial \tau_{t+1}}$ increases with the level of ability e^i . In other words, the higher the ability of the individual, the higher the indirect utility that he can reach when choosing his preferred level of taxation.

Proposition 6 *The equilibrium level of taxation in a majoritarian voting game is the following. If $F(\Psi^\tau) > \frac{1}{2}$, then $\tau^* = 0$. If $F(\Psi^\tau) < \frac{1}{2}$, then $\tau^* > 0$, with $\frac{\tau}{\varphi} = \frac{\bar{p}}{I}$.*

Our results suggest that, as long as the number of educated women is small, the tax-transfer scheme may not be supported as a politico-economic equilibrium. The society is trapped in a bad equilibrium: the high average costs of child care \bar{p} relative to the cost of providing care at home η , a low level of inherited human capital \bar{h} and a non-favorable attitude to women employment are associated with a high Ψ^τ . In such a society few women invest in education and even those who invest may find it convenient not to participate, with negative effects on growth. Instead, when there is a critical mass of educated women, the tax-transfer scheme can be politically supported.²⁰ As we showed before, this tax-transfer scheme has positive repercussions on the working of the economy.

5 Concluding Remarks

In a context of imperfect information on individual characteristics related to child rearing, high average child care costs reduce the incentives to invest in human capital and they may induce educated women not to participate. An institutional and cultural framework which is favorable to female employment may compensate for this individual heterogeneity and strengthen the incentives to invest in education and to work, with positive repercussions on growth.

In terms of policy implications, our analysis suggests that promoting an institutional and cultural environment in favor of women education and employment may generate economic benefits. We have identified a tax-transfer scheme which fulfils this purpose. Also

²⁰If women voted after their education decision has been taken, a critical mass of working women (rather than educated) would be required to observe a tax-transfer scheme in equilibrium. The timing of the voting game is consistent with our focus on education and with our set-up where investing in education is a pre-requisite to participate to the labour market.

the Lisbon strategy goes in this direction, identifying target values for European countries on female employment and on the institutional measures which may favor it (child care services, parental leaves etc). In particular, we have stressed that incentives to employment may also reinforce - through their effect on expectations - educational investments. Focusing on them may represent a good strategy to promote the macroeconomic performance of any country. Our results also suggest that the waste of talent arising when women do not invest in human capital or when educated women do not work may contribute to explain the low growth rates of the last decades of some European countries (Italy, for example). This is a perspective typically ignored by the empirical studies on the determinants of growth.

6 Appendix

6.1 Part A

We here show that, at any time t , when I is higher, the loss of human capital is lower, i.e. $\frac{\partial L}{\partial I} < 0$. From (19) we have that

$$\frac{\partial L}{\partial I} = \frac{\pi h_t \frac{\partial \left(\int_{\Psi}^{\Psi'} f(e) de \right)}{\partial I} \int_{\Psi}^1 f(e) de - \pi h_t \int_{\Psi}^{\Psi'} f(e) de \frac{\partial \left(\int_{\Psi}^1 f(e) de \right)}{\partial I}}{h_t \left(\int_{\Psi}^1 f(e) de \right)^2} \quad (38)$$

thus, $\frac{\partial L}{\partial I} < 0$ if the following condition is satisfied:

$$\frac{\partial \left(\int_{\Psi}^{\Psi'} f(e) de \right)}{\partial I} \int_{\Psi}^1 f(e) de - \int_{\Psi}^{\Psi'} f(e) de \frac{\partial \left(\int_{\Psi}^1 f(e) de \right)}{\partial I} < 0$$

Calculating the derivatives, the above condition becomes:

$$\left[f(\Psi') \frac{\partial \Psi'}{\partial I} - f(\Psi) \frac{\partial \Psi}{\partial I} \right] \int_{\Psi}^1 f(e) de - \left[-f(\Psi) \frac{\partial \Psi}{\partial I} \right] \int_{\Psi}^{\Psi'} f(e) de < 0$$

which can be rewritten as follows:

$$\frac{\partial \Psi}{\partial I} f(\Psi) \left[- \int_{\Psi}^1 f(e) de + \int_{\Psi}^{\Psi'} f(e) de \right] + f(\Psi') \frac{\partial \Psi'}{\partial I} \int_{\Psi}^1 f(e) de < 0$$

and finally as follows:

$$-\frac{\partial \Psi}{\partial I} f(\Psi) \int_{\Psi'}^1 f(e) de < -\frac{\partial \Psi'}{\partial I} f(\Psi') \int_{\Psi}^1 f(e) de$$

We already know from equations (11) and (13) that $\frac{\partial \Psi}{\partial I} < 0$ and $\frac{\partial \Psi'}{\partial I} < 0$. If f is uniform, $f(\Psi) = f(\Psi')$. Moreover, since $\Psi < \Psi'$ from (14), we have that $\int_{\Psi'}^1 f(e) de < \int_{\Psi}^1 f(e) de$. Thus, a sufficient condition for $\frac{\partial L}{\partial I} < 0$ is

$$\left| \frac{\partial \Psi}{\partial I} \right| < \left| \frac{\partial \Psi'}{\partial I} \right|$$

We now show that, when $\frac{\alpha \delta}{\alpha - \beta} = 1$ and $\delta = \gamma$, the above sufficient condition is always satisfied. Consider equations (23) and (24) and rewrite them as follows:

$$\Psi = \frac{\Sigma}{(1 - \frac{\bar{\rho}}{I})(1 - \frac{\beta}{\alpha})} \quad (39)$$

$$\Psi' = \frac{\Sigma}{(1 - \frac{\rho^H}{I})} \quad (40)$$

where

$$\Sigma = \nu \eta e$$

Differentiating (39) and (40), we obtain:

$$\left| \frac{\partial \Psi}{\partial I} \right| = \frac{\Sigma}{(1 - \frac{\beta}{\alpha})} \frac{\frac{\bar{\rho}}{I^2}}{(1 - \frac{\bar{\rho}}{I})^2}$$

$$\left| \frac{\partial \Psi'}{\partial I} \right| = \Sigma \frac{\frac{\rho^H}{I^2}}{(1 - \frac{\rho^H}{I})^2}$$

Simplifying and adjusting terms, we have that $\left| \frac{\partial \Psi}{\partial I} \right| < \left| \frac{\partial \Psi'}{\partial I} \right|$ when:

$$\left(\frac{I - \bar{\rho}}{I - \rho^H} \right)^2 \frac{\rho^H}{\bar{\rho}} > \frac{\alpha}{\alpha - \beta} \quad (41)$$

From condition (14) we know that $\left(\frac{I - \bar{\rho}}{I - \rho^H} \right) > \frac{\alpha}{\alpha - \beta}$. Since $\frac{\rho^H}{\bar{\rho}} > 1$, it is $\left(\frac{I - \bar{\rho}}{I - \rho^H} \right) > 1$ and therefore $\left(\frac{I - \bar{\rho}}{I - \rho^H} \right)^2 > \left(\frac{I - \bar{\rho}}{I - \rho^H} \right)$. Thus condition (41) is always satisfied. Therefore, $\left| \frac{\partial \Psi}{\partial I} \right| < \left| \frac{\partial \Psi'}{\partial I} \right|$, which guarantees that $\frac{\partial L}{\partial I} < 0$. Q.E.D.

6.2 Part B

We here show that the government budget constraint expressed by (27) guarantees that $\tau_{t+1} \leq \frac{\varphi_{t+1}\bar{\rho}}{I}$. To do this, we rewrite the government budget constraint in (27) as follows:

$$\begin{aligned} & \tau_{t+1} \left[\pi \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + (1 - \pi) \int_{\Psi_t}^1 w_{t+1}^i f(e) de + \hat{w}_{t+1} \int_0^1 f(e) de \right] \\ = & \left[\pi \varphi_{t+1} \frac{\rho^H}{I} + (1 - \pi) \varphi_{t+1} \frac{\rho^L}{I} \right] \left[\int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + \hat{w}_{t+1} \int_{\Psi'_{t+1}}^1 f(e) de \right] + \\ & (1 - \pi) \varphi_{t+1} \frac{\rho^L}{I} \left[\int_{\Psi_t}^{\Psi'_{t+1}} w_{t+1}^i f(e) de + \hat{w}_{t+1} \int_{\Psi_t}^{\Psi'_{t+1}} f(e) de \right] \end{aligned}$$

which can be rewritten as:

$$\tau_{t+1} \Lambda = \frac{\varphi_{t+1}\bar{\rho}}{I} \Delta + \frac{\varphi_{t+1}\rho^L}{I} (1 - \pi) \Omega$$

where

$$\Lambda = \pi \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + (1 - \pi) \int_{\Psi_t}^1 w_{t+1}^i f(e) de + \hat{w}_{t+1} \int_0^1 f(e) de$$

$$\Delta = \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + \hat{w}_{t+1} \int_{\Psi'_{t+1}}^1 f(e) de$$

and

$$\Omega = \int_{\Psi_t}^{\Psi'_{t+1}} w_{t+1}^i f(e) de + \hat{w}_{t+1} \int_{\Psi_t}^{\Psi'_{t+1}} f(e) de$$

and finally as follows:

$$\tau_{t+1} = \frac{\varphi_{t+1}\bar{\rho}}{I} \frac{\Delta}{\Lambda} + \frac{\varphi_{t+1}\rho^L}{I} (1 - \pi) \frac{\Omega}{\Lambda} \quad (42a)$$

Thus, we have that $\tau_{t+1} \leq \frac{\varphi_{t+1}\bar{\rho}}{I}$ when

$$\frac{\varphi_{t+1}\bar{\rho}}{I} \frac{\Delta}{\Lambda} + \frac{\varphi_{t+1}\rho^L}{I} (1 - \pi) \frac{\Omega}{\Lambda} \leq \frac{\varphi_{t+1}\bar{\rho}}{I}$$

which, after simple algebra, becomes:

$$\rho^L (1 - \pi) \Omega \leq \bar{\rho} (\Lambda - \Delta). \quad (43a)$$

Now we reintroduce the expressions for Ω , Λ and Δ to rewrite the above condition (43a) as follows:

$$\begin{aligned} & \rho^L(1 - \pi) \left[\int_{\Psi_t}^{\Psi'_{t+1}} w_{t+1}^i f(e) de + \widehat{w}_{t+1} \int_{\Psi_t}^{\Psi'_{t+1}} f(e) de \right] \\ \leq & \bar{\rho} \left[\begin{aligned} & \pi \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + (1 - \pi) \int_{\Psi_t}^1 w_{t+1}^i f(e) de + \\ & \widehat{w}_{t+1} \int_0^1 f(e) de - \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de - \widehat{w}_{t+1} \int_{\Psi'_{t+1}}^1 f(e) de \end{aligned} \right] \end{aligned}$$

Shifting all terms containing w_{t+1}^i on the left hand side and all members containing \widehat{w}_{t+1} on the right hand side the above condition becomes:

$$\begin{aligned} & \rho^L(1 - \pi) \int_{\Psi_t}^{\Psi'_{t+1}} w_{t+1}^i f(e) de - \bar{\rho}(1 - \pi) \int_{\Psi_t}^1 w_{t+1}^i f(e) de - \bar{\rho}\pi \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + \\ & \bar{\rho} \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de \tag{44} \\ \leq & \bar{\rho}\widehat{w}_{t+1} \int_0^1 f(e) de - \bar{\rho}\widehat{w}_{t+1} \int_{\Psi'_{t+1}}^1 f(e) de - \rho^L(1 - \pi)\widehat{w}_{t+1} \int_{\Psi_t}^{\Psi'_{t+1}} f(e) de \end{aligned}$$

Remembering that $0 < \Psi_t < \Psi'_{t+1} < 1$, after simple algebra the left hand side and the right hand side of the above condition (44) can be written respectively as

$$(\rho^L - \bar{\rho})(1 - \pi) \int_{\Psi_t}^{\Psi'_{t+1}} w_{t+1}^i f(e) de$$

and

$$\bar{\rho}\widehat{w}_{t+1} \int_0^{\Psi_t} f(e) de + [\bar{\rho} - \rho^L(1 - \pi)] \widehat{w}_{t+1} \int_{\Psi_t}^{\Psi'_{t+1}} f(e) de.$$

Since $\rho^L \leq \bar{\rho}$, the left hand side of (44) is (weakly) negative, while, since $\bar{\rho} \geq \rho^L(1 - \pi)$ the right hand side of (44) is (weakly) positive. Thus, condition (44) is always satisfied.

The government budget constraint in (27) guarantees that $\tau_{t+1} \leq \frac{\varphi_{t+1}\bar{\rho}}{I}$. Q.E.D.

6.3 Part C

If the government does not observe the true-cost type of women, the scheme it runs is characterized by the following budget constraint:

$$\begin{aligned} & \tau_{t+1} \left[\pi \int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + (1 - \pi) \int_{\Psi_t}^1 w_{t+1}^i f(e) de + \widehat{w}_{t+1} \int_0^1 f(e) de \right] \\ = & \varphi_{t+1} \frac{\bar{\rho}}{I} \left\{ \begin{aligned} & \pi \left[\int_{\Psi'_{t+1}}^1 w_{t+1}^i f(e) de + \widehat{w}_{t+1} \int_{\Psi'_{t+1}}^1 f(e) de \right] + \\ & (1 - \pi) \left[\int_{\Psi_t}^1 w_{t+1}^i f(e) de + \widehat{w}_{t+1} \int_{\Psi_t}^1 f(e) de \right] \end{aligned} \right\} \end{aligned} \quad (45)$$

It is immediate to see that $\tau_{t+1} < \varphi_{t+1} \frac{\bar{\rho}}{I}$. The individual budget constraints (28) and (29) become:

$$c_{t+1}^{iH} = (w_{t+1}^i + \widehat{w}_{t+1}) \left(1 - \frac{\rho^H}{I} - \tau_{t+1} + \varphi_{t+1} \frac{\bar{\rho}}{I} \right) \quad (46)$$

$$c_{t+1}^{iL} = (w_{t+1}^i + \widehat{w}_{t+1}) \left(1 - \frac{\rho^L}{I} - \tau_{t+1} + \varphi_{t+1} \frac{\bar{\rho}}{I} \right) \quad (47)$$

while (31) and (30) remain unchanged.

Simple algebra shows that the cut-off level of ability Ψ_t^τ in this case is equal to the one obtained at (34). The cut-off level of ability such that women find it profitable to work is instead given by:

$$\Psi'_{t+1}{}^{\tau'} \equiv \frac{\left[\widehat{w}_{t+1} (\eta - \varphi_{t+1} \frac{\bar{\rho}}{I}) \right]^{\frac{\alpha-\beta}{\alpha\gamma}}}{\left\{ \vartheta \nu^\delta \bar{h}_t^\delta \left[\frac{\beta}{\alpha} \vartheta \nu^\delta \bar{h}_t^\delta \left(1 - \frac{\bar{\rho}}{I} (1 - \varphi_{t+1}) - \tau_{t+1} \right) \right]^{\frac{\beta}{\alpha-\beta}} \left(1 - \frac{\rho^H}{I} - \tau_{t+1} + \varphi_{t+1} \frac{\bar{\rho}}{I} \right) \right\}^{\frac{\alpha-\beta}{\alpha\gamma}}} \quad (48)$$

with $\Psi'_{t+1}{}^{\tau'} > \Psi_{t+1}^\tau$ as defined in equation (36). The imperfect information of the government is such that it is less convenient for educated women to participate when compared with the case where the government can perfectly observe types. Yet, there still exists a range of parameter values under which some of the educated women abandon the labour market in the economy without tax and transfers, but they do not in the economy with tax and transfers and imperfect information. This range is now given by $\frac{\beta}{\alpha} \leq \frac{\rho^H}{I} - \frac{\bar{\rho}}{I} (1 - \frac{\beta}{\alpha}) \leq \frac{\beta}{\alpha} (1 - \tau_{t+1} + \varphi_{t+1} \frac{\bar{\rho}}{I})$.

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Figure 1a
Female upper secondary education and part-time employment

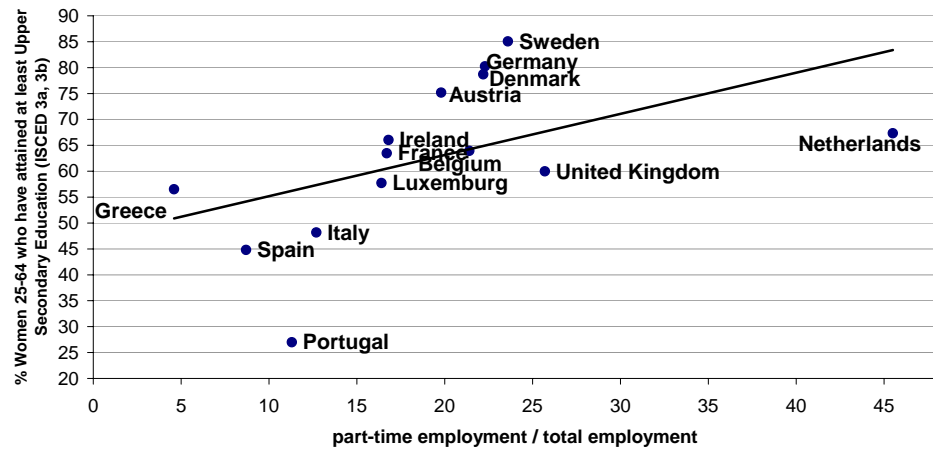


Figure 1b
Female upper secondary education and childcare services (0-2)

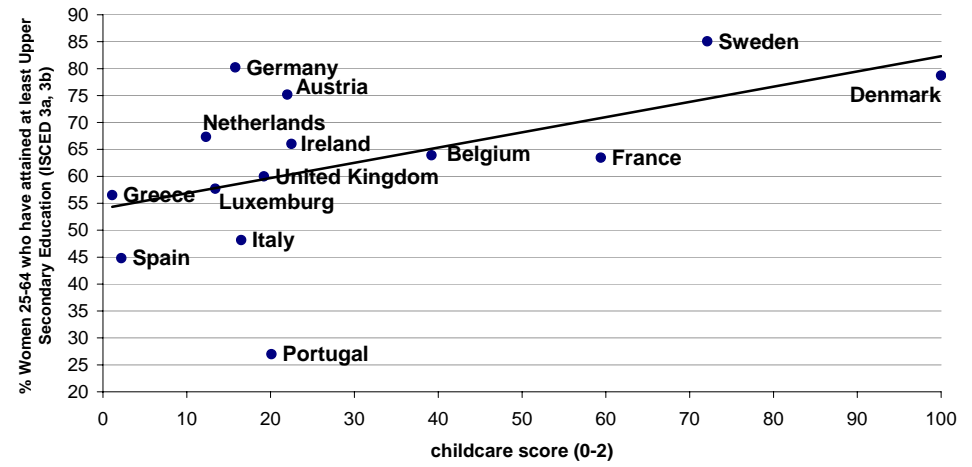


Figure 1c
Female upper secondary education and Family Expenditure %GDP

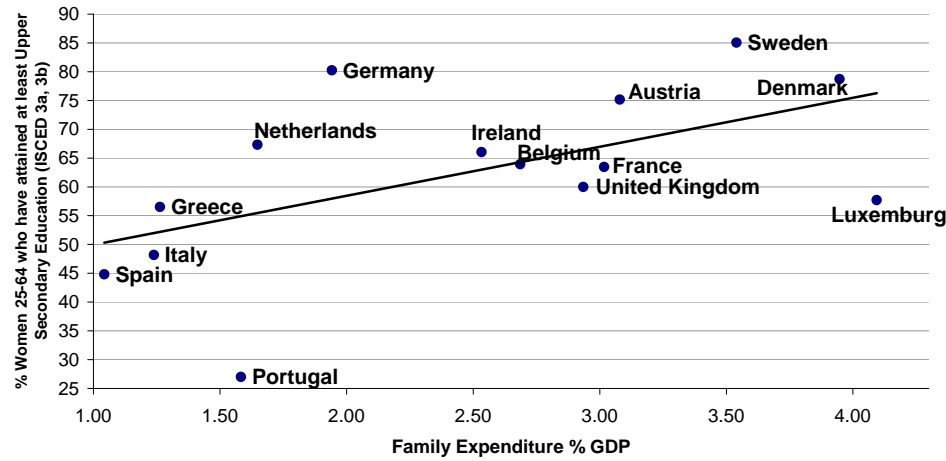


Figure 1d
Female upper secondary education and Culture Index

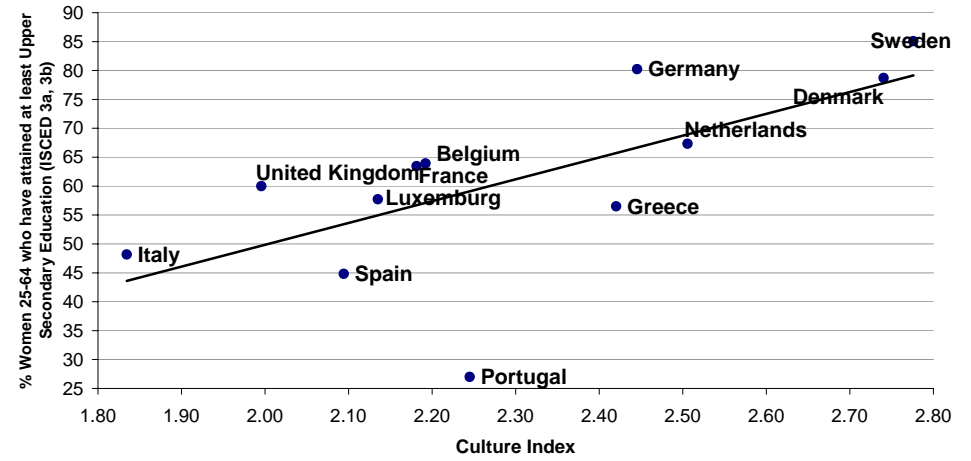


Figure 2a
Employed Female with upper secondary education and part-time employment

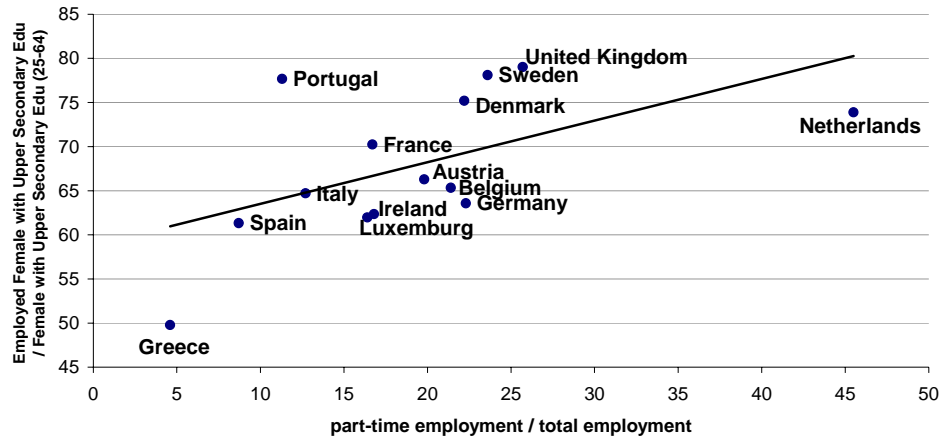


Figure 2b
Employed Female with upper secondary education and childcare (0-2)

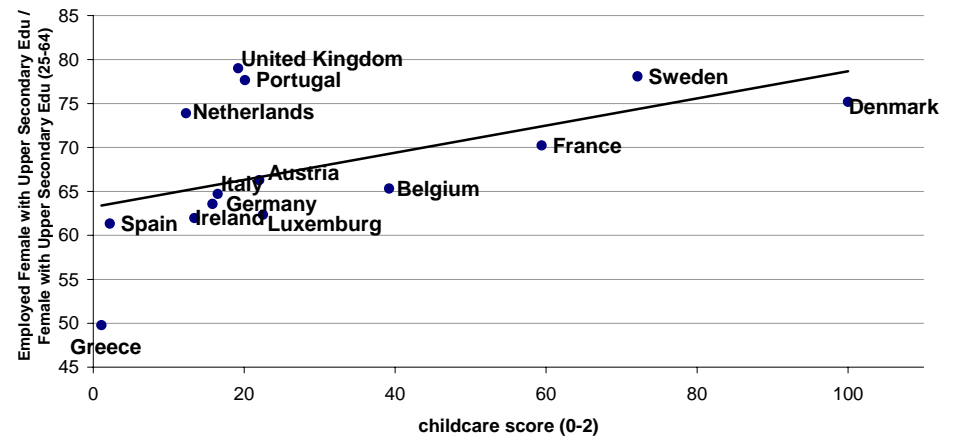


Figure 2c
Employed female with upper secondary education and Family Expenditure %GDP

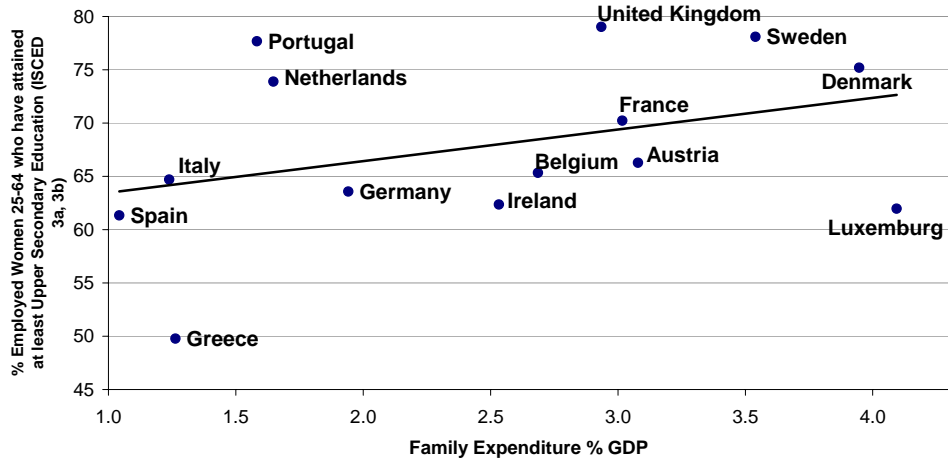


Figure 2d
Employed female with upper secondary education and culture index

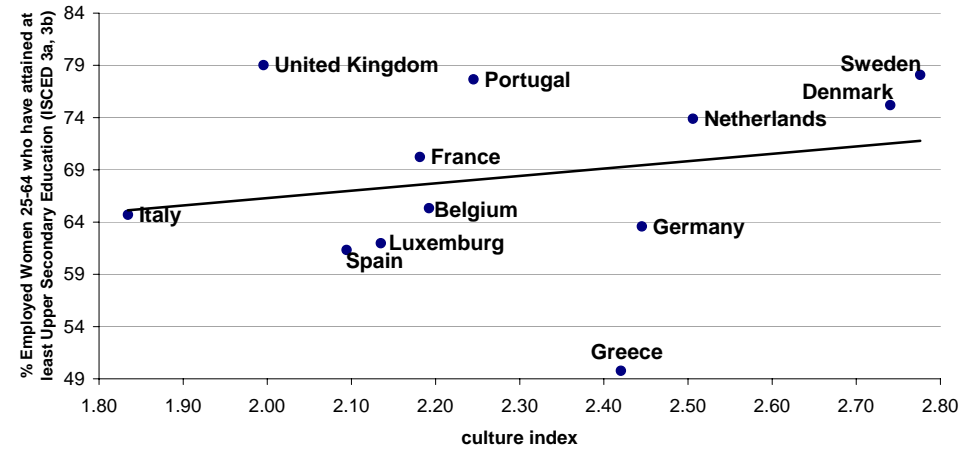
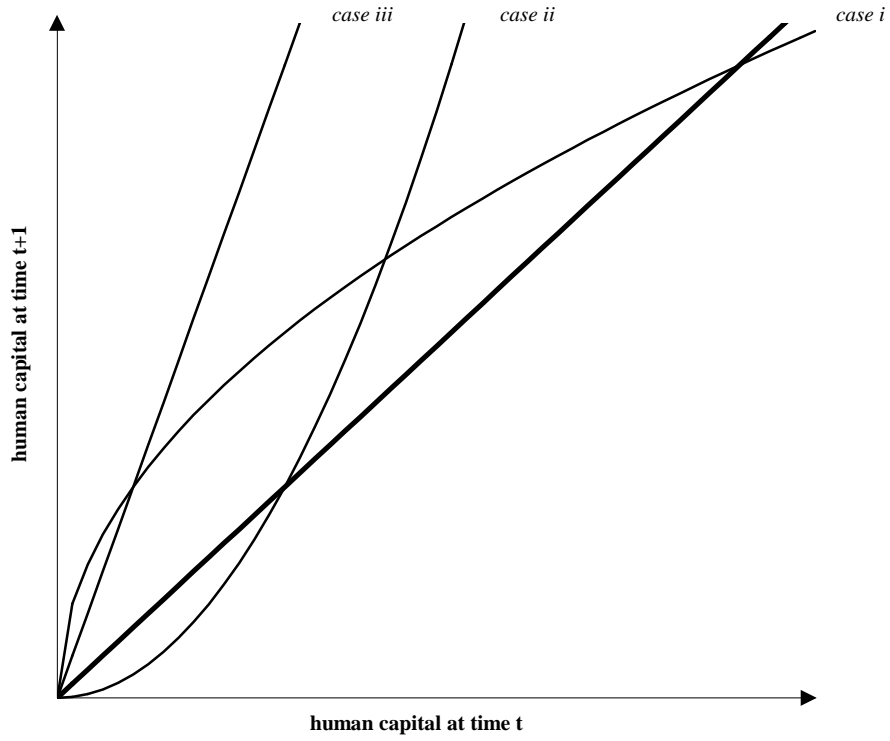


Figure 3
Human capital accumulation in a homogeneous household economy
 [case i, case ii, case iii]*



* case i: $\frac{\alpha\delta}{\alpha-\beta} < 1$; case ii: $\frac{\alpha\delta}{\alpha-\beta} > 1$; case iii: $\frac{\alpha\delta}{\alpha-\beta} = 1$